Programme

The Scientific programme of the Conference will be held in the Life Science Building, in Lecture Hall F015-016.

Friday

Chairman: Andrej Schinzel

14:00-14:40	Ákos Pintér	Győry 75: an outstanding scientific carreer
14:50-15:30	Endre Szemerédi	Maximum size of a set of integers with no two adding up to a square
15:30-16:00	Coffee Break	

Chairman: Robert Tijdeman

16:00-16:40	Jan-Hendrik Evertse	Root distance of polynomials
16:50-17:30	Yann Bugeaud	On the b-ary expansion of real numbers
19:00	Birthday Reception in the Main Building	

Saturday

Chairman: Vera T. Sós

9:00- 9:40	Andrzej Schinzel	On ternary integral recurrences
9:50-10:30	Attila Pethő	On multidimensional Diophantine approximation of algebraic numbers
10:30-11:00	Coffee Break	

Chairman: Cameron Stewart

11:00-11:40	Robert Tijdeman	Finite difference graphs of S-units
11:50-12:30	Lajos Hajdu	On the equation $1^k + \cdots + x^k = y^n$
12:30-14:00	Lunch	

Chairman: Jan-Hendrik Evertse

14:00-14:40	Cameron Stewart	S-unit equations and the abc conjecture
14:50-15:30	Mike Bennett	Computing elliptic curves over the rationals via Diophantine approximation
15:30-16:00	Coffee Break	

Chairman: Yann Bugeaud

16:00-16:40	Michel Waldschmidt	Effective upper bound for the solutions of a family of Thue equations involving powers of units of the simplest cubic fields

16:50-17:30	István Gaál	Power integral bases in algebraic number fields
17:40-18:20	Attila Bérczes	Effective results for Diophantine equations over finitely generated domains

Abstracts

Computing elliptic curves over the rationals via Diophantine approximation

Mike Bennett

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I will discuss new, old and older still methods for computing elliptic curves with bad reduction outside given sets of primes. Applying these, we are now able to find models for all elliptic curves over the rationals with prime conductor bounded by 10^{10} and, conjecturally, by 10^{12}. I will then mention extensions of these results to the case of more general conductors and to curves over number fields. This is a joint work with Andrew Rechnitzer.

Effective results for Diophantine equations over finitely generated domains

Attila Bérczes

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Let $A := \mathbb{Z}[z_1, \ldots, z_r] \supset \mathbb{Z}$ be a finitely generated integral domain over $\mathbb{Z}$ and denote by K the quotient field of A. Finiteness results

for several kinds of Diophantine equations over A date back to the middle of the last century. S. Lang generalized several earlier results on Diophantine equations over the integers to results over A, including results concerning unit equations, Thue-equations and integral points on curves. However, all his results were ineffective.

The first effective results for Diophantine equations over finitely generated domains were published in the 1980's, when Győry developed his new effective specialization method. This enabled him to prove effective results over finitely generated domains of a special type.

In 2011 Evertse and Győry combined the method of Győry with a recent result of Aschenbrenner which made it possible to prove effective results for unit equations $ax + by = 1$ in $x, y \in A^*$ over arbitrary finitely generated domains A of characteristic 0. Later Bérczes, Evertse and Győry obtained effective results for Thue equations, hyper- and superelliptic equations and for the Schinzel-Tijdeman equation over arbitrary finitely generated domains. Bérczes proved effective results for equations $F(x, y) = 0$ in $x, y \in A^*$ for arbitrary finitely generated domains A, and for $F(x, y) = 0$ in $x, y \in \overline{\Gamma}$, where $F(X, Y)$ is a bivariate polynomial over A and $\overline{\Gamma}$ is the division group of a finitely generated subgroup Γ of K^*.

In the talk a survey of the results obtained by the specialization method of Győry and Evertse will be presented.

On the b-ary expansion of real numbers

Yann Bugeaud

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It is commonly believed that real numbers like $\sqrt{2}, \mathrm{e}, \log 2, \ldots$ are normal to every integer base. This challenging open problem seems to be at present completely out of reach. We focus our attention to apparently simpler questions : we take a point of view from combinatorics on words and regard their b-ary expansions as infinite words written over the alphabet $\{0, 1, \ldots, b-1\}$. We survey recent results showing that, for e, log(2016/2015), badly approximable numbers and algebraic numbers (among other classical numbers), these infinite words are not 'too simple', in a suitable sense.

Root distance of polynomials

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Let $f \in \mathbb{Z}[X]$ be a non-zero polynomial of degree $n \geq 2$. Suppose that f factors over $\mathbb{C}$ as $a\prod_{i=1}^{n}(X-\alpha_i)$. Define $\mathrm{sep}(f) := \min_{i,j}|\alpha_i-\alpha_j|$ where the minimum is taken over all pairs i, j with $1 \leq i < j \leq n$, and let $H(f)$ denote the maximum of the absolute values of the coefficients of f. From an elementary inequality due to Mahler it follows that

$$\mathrm{sep}(f) \geq c(n)H(f)^{1-n},$$

where $c(n)$ is a positive number depending only on $n = \deg f$. It can be shown that in terms of $H(f)$ this is best possible if $n = 3$. On the other hand, using Baker's method one can show that

$$\mathrm{sep}(f) \geq c'(n)H(f)^{1-n}(\log 2H(f))^{1/(10n-6)}$$

if $n \geq 4$, where $c'(n)$ is effectively computable and depends on n only. We will discuss this and more general results.

Power integral bases in algebraic number fields

István Gaál
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This talk is a survey about classical and new results on monogenity of number fields and calculating generators of power integral bases. We consider this question in the absolute and in the relative case, as well. We mention algorithms in specific number fields and statements on infinite parametric families of number fields. Many of the results are connected with the work of Kálmán Győry.

On the equation $1^k + \cdots + x^k = y^n$

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Coauthors: A. Bérczes, T. Miyazaki, I. Pink

For positive integers k and x, write

$$S_k(x) = 1^k + \cdots + x^k$$

for the sum of the k-th powers of the first x positive integers. In the talk we consider the Diophantine equation

$$S_k(x) = y^n \tag{1}$$

in positive integers k, n, x, y with $n \geq 2$. The equation has a long history, going back to Lucas, Watson and others. In 1956 Schäffer proved that for any fixed $(k, n) \neq (1, 2), (3, 2), (3, 4), (5, 2)$, equation (1) has only finitely many solutions. Further, he conjectured that for (k, n) as above, the equation has the only nontrivial solution $(k, n, x, y) = (2, 2, 24, 70)$.

Since then, many mathematicians have obtained interesting theorems in this topic. In the first part of the talk we survey the related literature, and mention some of the most important results, due to Bennett, Győry, Jacobson, Pintér, Tijdeman, Voorhoeve, Walsh and others.

In all the known results, the parameter k, and sometimes also n, are assumed to be fixed. In the second part of the talk we present new results concerning equation (1) under some assumptions on x, letting k, n, y be completely free. We give a result saying that if x satisfies certain congruence conditions, then (1) has only solutions known already, and we also present the complete solution of (1) for $x < 25$ with $n > 2$. These results verify the conjecture of Schäffer for the corresponding ranges of the parameters.

On multidimensional Diophantine approximation of algebraic numbers

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Coauthors: Csanád Bertók, Michael Pohst

In this talk I report on a joint paper with Michael Pohst and Csanád Bertók. We developed an algorithm for solving the dual problems of approximating linear forms and of simultaneous approximation in number fields F. Using earlier ideas for computing independent units by Buchmann, Pethő and later Pohst we construct sequences of suitable modules in F and special elements β contained in them. The most important ingredient in our methods is the application of the LLL-reduction procedure to the bases of those modules. For LLL-reduced bases we derive improved bounds on the sizes of the basis elements. From those bounds it is quite straight-forward to show that the sequence of coefficient vectors $(x_1, ..., x_n)$ of the presentation of β in the module basis becomes periodic. We can show that the approximations which we obtain are close to being optimal. Thus our algorithm can be considered as such a generalization of the continued fraction algorithm which is periodic on bases of real algebraic number fields.

Győry 75: an outstanding scientific carreer

Ákos Pintér

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In this lecture a personalized survey about the greatest moments and milestones of Kálmán Győry's scientific life is given.

On ternary integral recurrences

Andrzej Schinzel

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Theorem. *Every essentially ternary integral recurrence* u_n*, the companion polynomial of which has a double zero,there exists an integer* $D > 0$ *such that for all integers* m *prime to* D *infinitely many terms* u_n *are divisible by* m*.*

S-unit equations and the abc conjecture

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In this lecture we shall discuss a refined version of the abc conjecture which we formulated with Robert and Tenenbaum. In addition we shall discuss the link with S-unit equations and the work of Győry and Yu on the abc conjecture in number fields.

Maximum size of a set of integers with no two adding up to a square

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Coauthors: Simao Herdade, Ayman Khalfallah

Erdős and Sárközy asked the maximum size of a subset of the first N integers with no two elements adding up to a perfect square. In this talk we prove that the tight answer is $\frac{11}{32}N$ for sufficiently large N. We are going to prove some stability results also.

On finite difference graphs of S-units

Robert Tijdeman

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Coauthors: Kálmán Győry, Lajos Hajdu

Let K be a given algebraic number field and S a given finite set of places of K including all infinite ones. We deal with finite graphs G where the vertices have values in K and two vertices are connected by an edge if and only if their values differ by an S-unit. We call such graphs difference graphs. We distinguish between equivalence classes of difference graphs and between infinitely representable and finitely representable equivalence classes of difference graphs. We present two theorems on the representability of the union of two difference graphs of which one is finitely representable which generalize some older results of Kálmán Győry.

Effective upper bound for the solutions of a family of Thue equations involving powers of units of the simplest cubic fields

Michel Waldschmidt

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Coauthors: Claude Levesque

E. Thomas was one of the first to solve an infinite family of Thue equations, when he considered the forms $F_n(X,Y) = X^3 - (n-1)X^2Y -$

$(n+2)XY^2 - Y^3$ and the family of equations $F_n(X,Y) = \pm 1$, $n \in \mathbb{N}$. This family is associated to the family of the simplest cubic fields $\mathbb{Q}(\lambda)$ of D. Shanks, λ being a root of $F_n(X,1)$. We introduce in this family a second parameter by replacing the roots of the minimal polynomial $F_n(X,1)$ of λ by the a-th powers of the roots and we effectively solve the family of Thue equations that we obtain and which depends now on the two parameters n and a.